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Measurements of the Effective Cesium-Sodium Ion Exchange Rate in IONSIV® IE-910 and IE-911

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Abstract

We measured the effective rate of sodium-cesium ion exchange for both IONSIV® IE-910 and IE-911 in "Average" Savannah River Site simulant. The measured values were 2.7 – 5.8 x 10^{-18} m^2/s for IE-910 and 2.5 – 5.8 x 10^{-11} m^2/s for IE-911. The difference in rate of ion exchange between IE-910 and IE-911occurs due to differences in density and particle size, and partial permeation of cesium into IE-911. Analysis of the water content in IE-910 by thermogravimetric techniques yielded an effective cesium diffusivity of 1.75 x 10^{-19} m^2/s.

Introduction

Researchers recently examined ion for cesium removal from high-level waste at the Savannah River Site (SRS). The proposed process uses IONSIV® IE-911 (UOP LLC, Molecular Sieves Division, Des Planes, IL) — the engineered form of crystalline silicotitanate (or IONSIV®IE-910) — as the sorbent for cesium . The rate at which ions exchange is one of the key transport parameters of this process. Examination of the literature on crystalline silicotitanate (CST) reveals a significant amount of work has been done in this subject matter[1, 2].

A typical method for measuring diffusivity involves monitoring the concentration of cesium in solution after contacting the granules of IE-911 for a period of time. Successful application of the experiment requires high precision determination of the cesium concentration in solution such as accomplished with gamma counting techniques.

Another approach involves directly measuring the amount of cesium loaded in the granules. One can measure the sorbed cesium by the following approaches. X-rays diffraction techniques can detect minute changes in the unit cells as the cesium ions are accommodated. Nuclear Magnetic Resonance of cesium (cesium has a quadrupolar nuclei) can directly detect the amount of cesium in the granules. Infrared spectroscopy detects the shift in the Si-O or Ti-O vibrational stretch when the cesium ion enters the IE-910 unit cells. Thermogravimetric analysis detects the water loss associated with sodium ions. We chose to monitor cesium loading with thermal analysis (TGA) and spectroscopic (infrared) techniques. These techniques require very little sample preparation. In this work, we examined the application of thermogravimetric analysis and infrared spectroscopy on cesium loading on granular material (like IE-910 and IE-911).

A third way of estimating cesium diffusivity involves by adjusting the cesium diffusivity in water for viscosity in salt solution, geometry (depending on the pore geometry model) and absorption in the engineering granules. However, as Latheef et al.[2] clearly state, measuring ionic diffusivity in engineering materials proves extremely difficult due to the complexity of today's material. In the case of CST, its engineering form is a mixture of CST (IE-910) and a porous binder (zirconium hydroxide). See Figure 1 for a schematic. The two phases have different resistances to cesium transport. Since the binder phase has a much bigger pore size (38 Å average pore radius based on BET measurements)[3] one expects the resistance from IE-910 (average pore radius is 4 Å or less based on XRD) to dominate. In addition, most of the cesium absorption occurs in IE-910. Another problem involves finding the true effective distance traveled by the cesium ions.

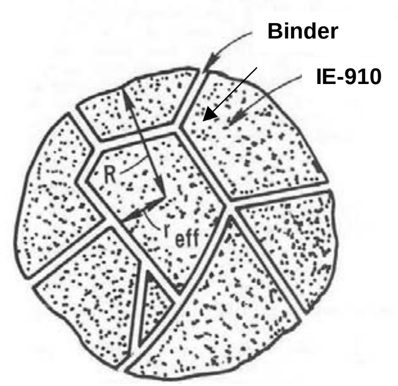

Figure 1. Schematic picture of IE-911

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[1] D. Gu et al., "Cs^+ Ion Exchange Kinetics in Complex Electrolyte Solutions Using Hydrous Crystalline Silicotitanates," Ind. Eng. Chem. Res. 1997, 36, 5377-5383.
[2] I. M. Latheef, M. E. Huckman and R. G. Anthony, "Modeling Cesium Ion Exchange on Fixed-Bed Columns of Crystalline Silicotitanate Granules," Ind. Eng. Chem. Res. 2000, 39, 1356-63.
[3] W. R. Wilmarth et al. "Examination of Pre-Production Samples of UOP IONSIV® IE-910 and IE-911," WSRC-TR-2001-00221, Revision 0, April 18, 2001.

This report presents the measured rate of exchange between cesium and sodium in IE-910 and IE-911 granules in "Average" SRS simulated waste.[4] The rest of the report interchangeably uses the terms "effective diffusion" with "ion exchange rate". Typically, "effective diffusivity" is used to describe the permeation of a solute into a solvent. For example, hydrogen diffusion into metals. The term "ion exchange rate" is used to describe the transport of one ion leaving the resin and a different but of the same charge ion moving into the resin. The researcher is using mathematical modeling derived for "effective diffusivity" to described ion exchange rate in IE-911. There is no loss of generality in using both terms to describe cesium loading into IE-911. The testing used several techniques for measuring cesium in IE-910 and IE-911 to establish consistency of the data. Some of the testing with IE-910 granules used different starting cesium concentrations to determine if film formation rate limits cesium adsorption. Testing also looked at the infrared and gravimetric analysis of the granules as a function of contact time with a solution with composition similar to SRS "Average" but with high cesium levels (50 mM). The high cesium level overwhelms the influence of film formation on the cesium adsorption rate.

Experimental Section

The authors contacted about 0.1 grams of IE-911 (batch # 89991000009) or IE-910 (batch # 30050-48) was contacted with 20 mL of "Average" salt solution for different length of times (with the solution containing trace levels of Cs^{137}). During the contact test the slurry shook in an orbital shaker (300 rpm) at 25 °C. At the end of the test, personnel filtered the slurry with a 0.02 microns polyester filter paper (in a dead end configuration). The researchers determined the cesium content of the filtrate using an in-house gamma counter. We performed an identical experiment using "Average" simulant with no radioactive cesium tracer. We analyzed the IE-911 granules from this experiment using thermal analysis techniques (Thermogravimetric Analysis from T. A. instruments) and by spectroscopic techniques (Nicolet 210).

Results and Discussion

Cesium Diffusivity in IONSIV® IE-911 and IE-910

Figure 2 shows the cesium concentration results of contacting Average salt simulant with IE-910 or IE-911 as a function of time as determined by gamma counting. The figure depicts three sets of data. Two of the three curves correspond to IE-910 in different starting cesium concentration. The other curve corresponds to Average salt simulant in contact with IE-911. The points in Figure 2 represent averages of duplicate testing.

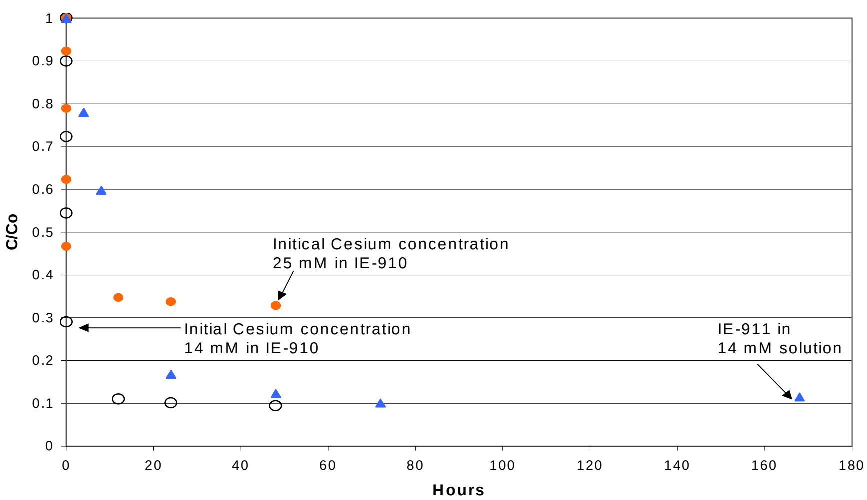

Figure 2. The temporal evolution of the cesium concentration in an "Average" solution in contact with IE-911 and/or IE-910.

[4] D. D. Walker, "Preparation of Simulated Waste Solutions," WSRC-TR-99-00116, Rev. 0, March 15, 1999.

Looking at Figure 2, the initial cesium concentration had minimal apparent effect on the rate of cesium loading on IE-910 was seen. All the curves in Figure 2 can not be approximated with a single exponential decay. We derived the apparent cesium-sodium ion exchange rate from the initial part data, from time less than 2 hours, using a series solution derived from the literature[5] for a well-stirred solution of limited volume. This model is called "Homogeneous Single Phase Model" for powders.

$$\frac{M_t}{M_\infty} = 1 - \sum_{n=1}^{\infty} \frac{6\alpha(\alpha+1) \times e^{-\frac{D q_n^2 t}{a^2}}}{9 + 9\alpha + q_n^2 \alpha^2}; \quad \alpha = \frac{\text{Volume of fluid}}{\text{Volume of CST} \times K_d \times \rho}; \quad \tan q_n = \frac{3q_n}{3 + \alpha q_n^2}; \quad 1)$$

In this expression, M_t is the amount of cesium loaded on the resin, D is the cesium diffusivity, "a" stands for the particle radius, and M_∞ is the maximum amount of cesium the resin can load. The symbol, K_d, stands for the distribution coefficient (L/mg) and ρ is the resin density.

The expression above does not account for the effect of the film formation outside of the granules during the shaking test. The film slows down cesium transfer into the granules. In order to use the above expression, the ordinate scale in Figure 2 must be transformed to another scale (for example M_t/M_∞) with the following expression.

$$\frac{M_t}{M_\infty} = \frac{1 - \frac{C}{C_o}}{1 - \frac{C_\infty}{C_o}} \quad 2)$$

Figure 3 shows the effect of this transformation on the data from Figure 2.

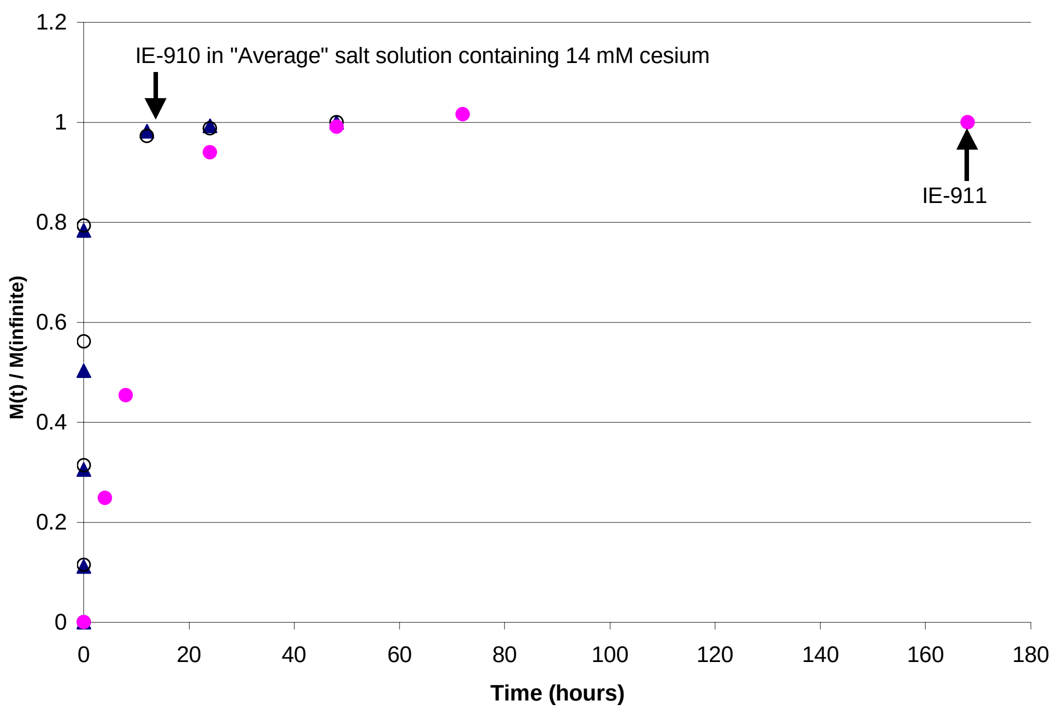

Figure 3. The amount of cesium loaded on to IE-910 and 911 as a function of contact time.

[5] P. C. Carman and R. A. Haul, "Measurement of Diffusion Coefficients," Proc. Roy. Soc. A, 1954, 222, pp. 109-118

Figure 4 provides a more detailed plot of cesium loading (from Figure 3) onto IE-910. In this figure, two data sets provide the output from the equation 1. The third data set provides the experimental results obtained from the cesium-loading test. The error bars in Figure 4 derive from duplicate testing. The value of the upper portion of an error bar reflects the highest value obtained and the lower portion of the error bar represents the lowest value obtained in that test. Note the model did not fit the data over the whole time domain. This behavior results from a change in diffusion rate with time. We chose to fit the data by regions. One curve from the model best approximated the data set at the highest cesium loading. The other curve best fit the data in the early loading. The particle size for IE-910 powder is 0.1 microns and for the IE-911 granules is 300 microns.

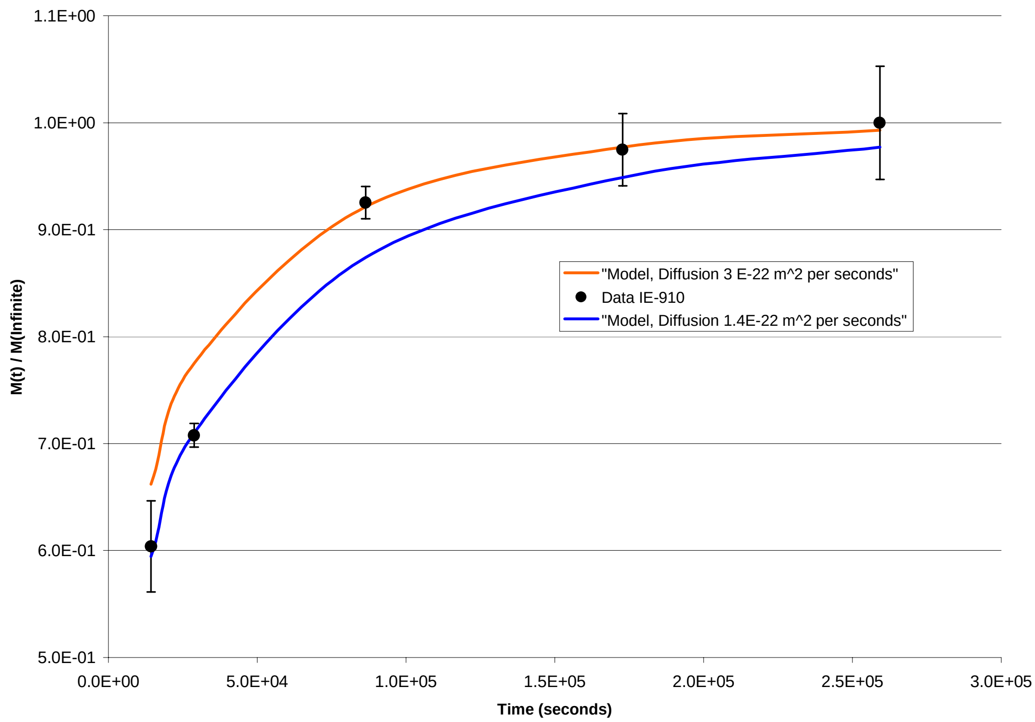

Figure 4. The cesium loading on the IE-910 powders. The two curves are output from equation 1.

We performed least square fitting of the data in Figure 4 using the prediction of the diffusion equation. The fitting consisted of minimizing the sum of the square of the difference between the data and the prediction with diffusivity as a variable. We searched for the best value that minimized the sum of the square difference. The fitting yielded the diffusivity value that best approximates the kinetics of the curves in Figure 4. The resulting apparent cesium diffusivity in IE-910 during the early test equaled 1.4 x 10^{-22} m^2/s and increased to 3 x 10^{-22} m^2/s at the end of the test. The effective cesium diffusivity was calculated from the following expression.

$$D_{apparent} = \frac{D_{effective}}{\left(1 + \frac{[1-\varepsilon] \times K_d \times \rho}{\varepsilon}\right)} \quad ; \qquad 3)$$

In this equation, the symbol "K_d" stands for the cesium distribution coefficient (usually 2100 mL/g), "ρ" stands for IE-910 or IE-911 density (2.92 g/mL for IE-911 and 3.01 g/mL for IE-910), and "ε" stands for the porosity in IE-910 or IE-911 (porosity is about 0.24 for IE-910 and 0.55 for IE-910). These values were substituted into equation 3. The effective cesium diffusivity in IE-910 is **2.7 to 5.8 x 10^{-18} m^2/s**. The rapid (within minutes) loading of the IE-910 particles occurs due to their small size.

Figure 5 shows the cesium loading on the IE-911 granules. Again, the model did not fit the loading curve over the whole time domain. We fitted the data in Figure 5 with two curves. One of the curves fit the data obtained early in the test. That curve yielded an apparent cesium diffusivity of 1.3×10^{-15} m^2/s. The other curve fitted the data obtained later in the test. The calculated apparent diffusivity from this curve was 3×10^{-15} m^2/s. The corresponding effective diffusivity (after substituting into equation 3) is **2.5 to 5.8 x 10^{-11} m^2/s**.

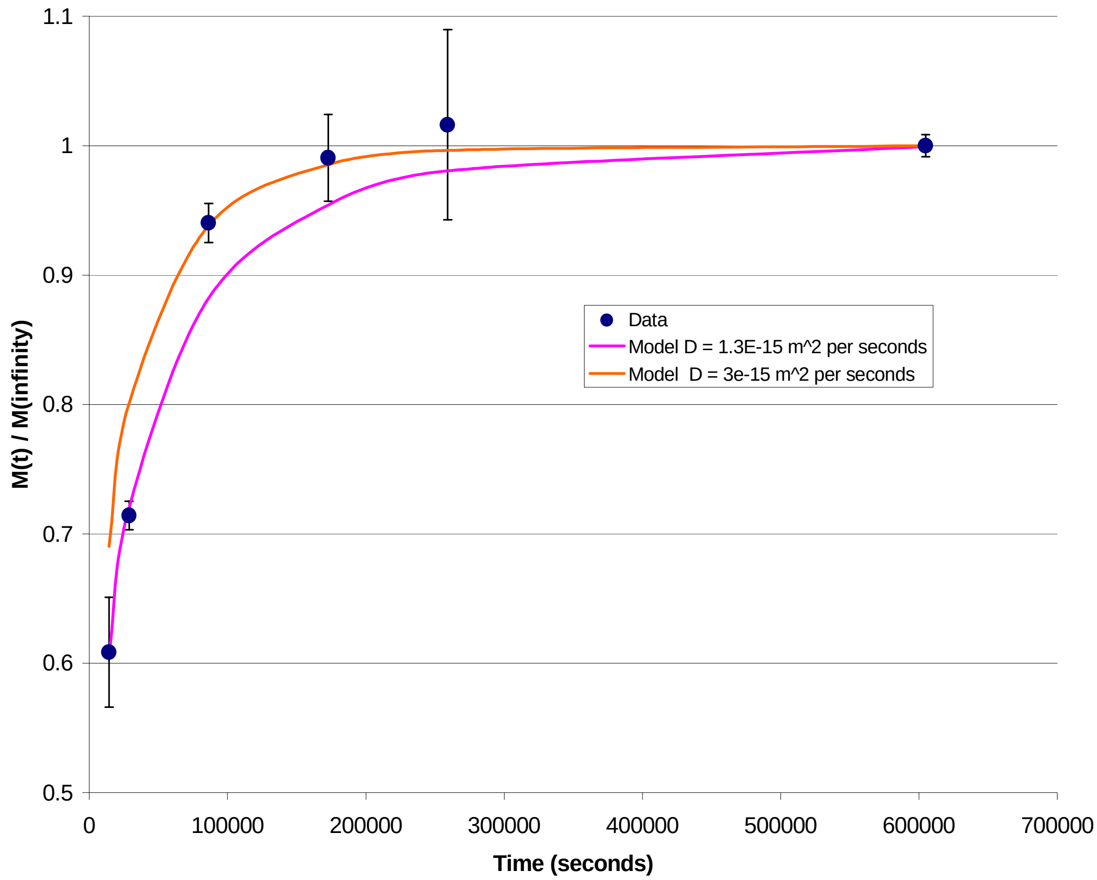

Figure 5. The cesium loading on IE-911 granules. The two smooth curves are the output from equation 1.

The authors have no explanation for the lower cesium diffusivity early in the batch test.
Table 1 compares the cesium diffusivity estimated by several research teams. As can be seen from Table 1, all previous studies report a cesium diffusivity value between 2 and 8 x 10^{-11} m^2/s. This study reports similar values. Examination of Table 1 reveals that analyzing column output data or conducting batch tests yielded similar cesium diffusivity values.

Table 1. Comparison of the cesium diffusivity values in IE-911obtained by different methods.

D (m^2/s)	Method	Simulant	Reference
7.6 x 10^{-11}	Calculation	Average SRS	R. G. Anthony[1]
2.5 x 10^{-11}	Fitting column output	Average SRS	M. E. Huckman[2]
2-7 x 10^{-11}	Batch tests	Simplified Simulant	M. E. Huckman[3]
8.3 x 10^{-11}	Fitting column output	Average SRS	T. Hang[4]
2.5 - 5.8 x 10^{-11}	Batch Tests	Average SRS	This work

[1]R. G. Anthony, "Crystalline Silicotitanate Ion Exchange Support for Salt Alternatives," Final report for contract #KF-90594-O, September 21, 2001.
[2]M. E. Huckman et al. "Treating Savannah River Waste Using UOP IONSIV IE-911, Revised Design Based on Laboratory Column Experiments," September 25, 1998.
[3] M. E. Huckman et al. " Mathematical Modeling of Ion Exchange Columns with Emphasis on Hydrous Crystalline Silico-Titanates, Molecular Sieves and Non Ideal Solutions," in Emerging Separation Technologies for Metals II, Edited by R. G. Bautista, The Minerals, Metals and Materisal Society, pp. 141-157, 1996.
[4]T. Hang and R. A. Dimenna, "Speedup™ Ion Exchange Column Model," WSRC-TR-99-00401, February 16, 2000.

Explanation for the Different Cesium Diffusivity in IE-910 and IE-911

One observation of this work is that the cesium diffusivity in IE-910 measured three to four orders of magnitude lower than in IE-911. As previously stated the IE-911 granule is composed of IE-910 and a porous binder (zirconium hydroxide). Cesium diffusion through the binder occurs rapidly since the binder has a large pore diameter (37 microns). Therefore, the rate of cesium loading in IE-911 should roughly equal that in IE-910. However, the data indicate the cesium loading rate in IE-911 is several orders of magnitude faster than in IE-910. One possible explanation is that the binder irreversibly absorbs cesium. This absorption can account for the initial fast decrease of cesium concentration in solution. But the absorption due to the binder is minimal. Another explanation is that cesium permeated only a few microns into the IE-911 particles. Therefore, the apparent diffusivity will be a large number. No test was conducted to verify this hypothesis. Yet another possibility is that the particle size of IE-911 is larger than the IE-910 powder as shown below. The equations below are derive from the diffusivity is proportional to the square of the distance and inversely proportional to time of the experiment. The diffusivity was adjusted for cesium absorption in IE-910 and IE-911 (notice the K_d term in the equations).

$$t_{observed} = \frac{K_d\ \rho_{910}\ R_{910}^2}{D_{910}}\ ; \quad \text{but also } t_{observed} = \frac{K_d\ \rho_{911} R_{911}^2}{D_{911}} \quad \text{take the ratio}$$

$$\frac{D_{911}}{D_{910}} = \frac{\rho_{911}\ R_{911}^2}{\rho_{910}\ R_{910}^2} = 0.85 \times \frac{R_{911}^2}{R_{910}^2};\ R_{911} \text{ is much larger than } R_{910}$$

From the expression above the symbol "K_d" stands for the distribution coefficient, "ρ" stands for the density and "*R*" stands for the radius of the particle. As can be seen from this expression, cesium diffusivity is faster in IE-911 than in IE-910 due to a larger particle size and density in IE-911. Our experimental results proved this prediction correct. Although the measured diffusivity seemed to agree with previous measurements[6] the authors felt a new way of measuring cesium loading on IE-910 and 911 was needed.

[6] M. E. Huckman, D. Gu, C. V. Philip and R. G. Anthony, "Mathematical Modeling of Ion Exchange Column with Emphasis on Hydrous Crystalline Silico-Titanate, Molecular Sieve Non-Ideal Solutions," in

Cesium Diffusivity Determination from Thermogravimetric Analysis

In a previous report, the amount of water associated with the sodium ions in IE-910 was measured[3]. As the cesium exchanges with sodium ions less water loss at 220 ºC is seen. The researchers measured the water weight loss of IE-911 after contacting an Average salt solution (14 mM cesium concentration) for different length of times. The results are shown in Figure 6 and 7. As can be seen from both figures, except for curve labeled “Test 38”, the amount of water at 220 °C decreases with contact time in average salt solutions. This is clearly shown in Figure 8. The data presented in Figure 8 is normalized relative to the amount of water in the as received IE-911. The fitting curve in Figure 8 was not used to calculate the diffusivity but it functions to aid the eye.

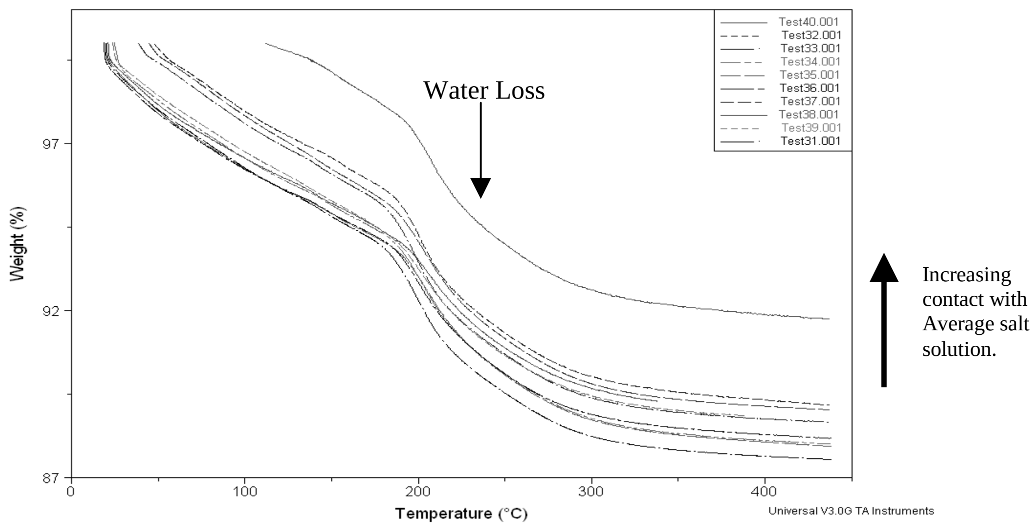

Figure 6. The weight loss of IE-911 with temperature after contacting average salt solution for 10 sec, 20 sec, 40 sec, 2 min, 5 min, 10 min, 20 min, 1 hr, 3 hr, and 5hr.

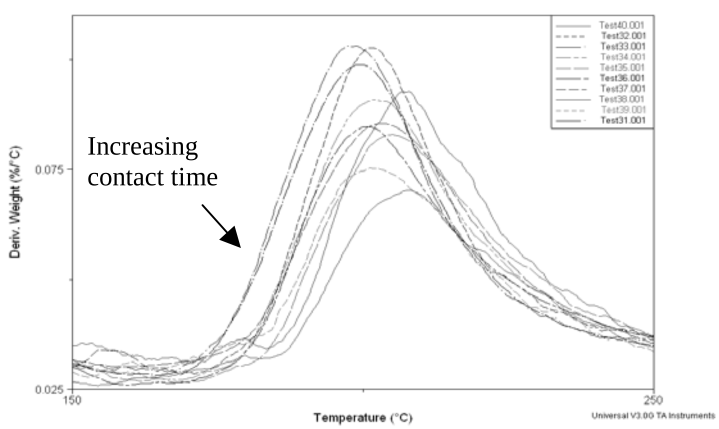

Figure 7. The derivative of the curves in Fig. 6 with temperature.

Emerging Separation Technologies for Metals II, The Mineral, Metals & Material Society, 1996, pp. 143-155.

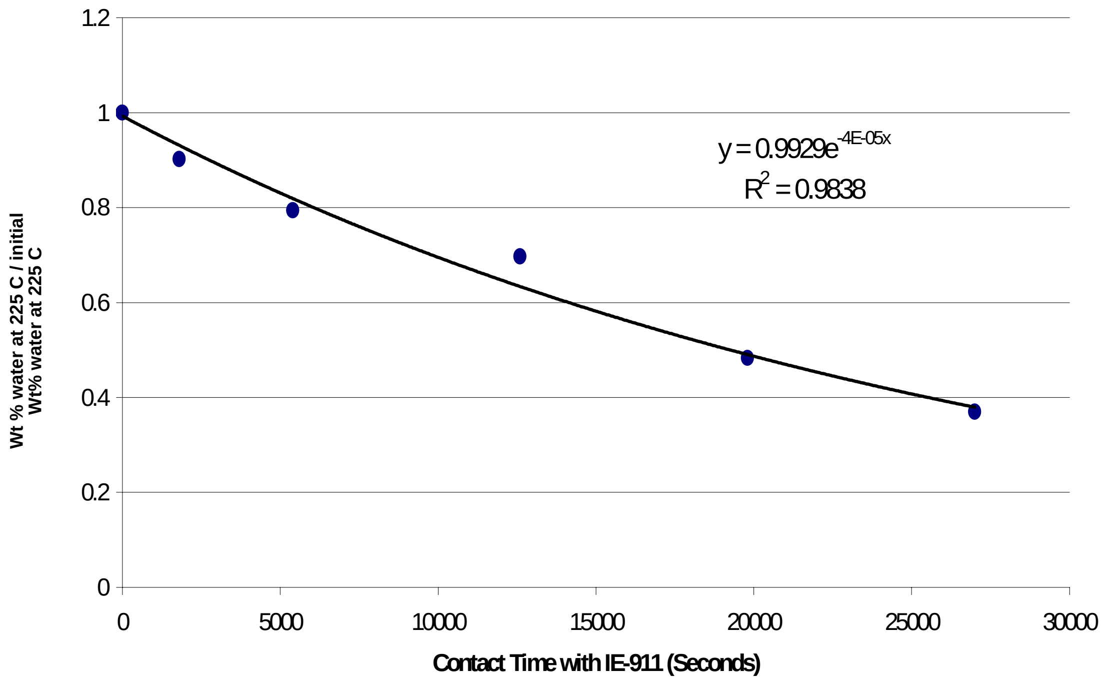

Figure 8. The water loss of IE-911 after contacting an Average salt solution for different times. The initial cesium concentration was 14 mM.

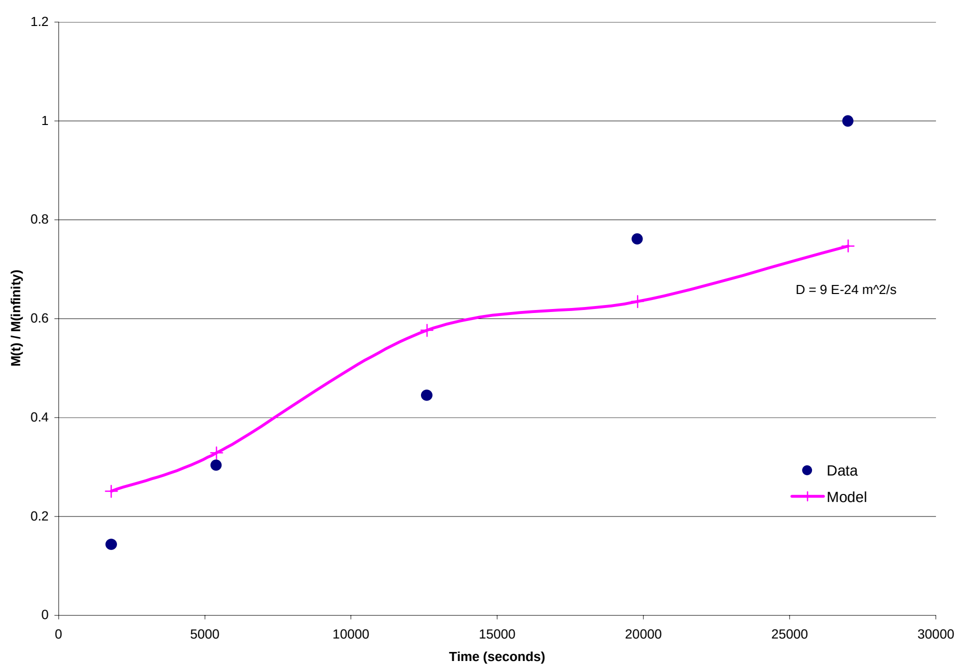

Figure 9. Transformation and fitting of the data in Figure 8.

The data in Figure 8 was transformed according to Equation 2 and presented in Figure 9. Included in Figure 9 is the output from equation 1 that best approximates the data in Figure 9. Although the degree of fitness is small ($r^2 = 0.72$), the calculated apparent diffusivity from the data in Figure 8 is 9×10^{-24} cm^2/s. The corresponding effective diffusivity is **1.75×10^{-19} m^2/s**. This value is one order of magnitude lower than the value obtained from the measurements of the solution tests.

Cesium Diffusivity Determination from Infrared Spectroscopy

Portions of the cesium loaded IE-911 samples were analyzed by Infrared spectroscopy. The spectral analysis of the samples is shown in Figure 10. Vertical lines were drawn through the maximum absorption of each peak. Looking at Figure 10, it can be clearly seen that none of the peaks shift significantly and no trend is discernable with contact time in average salt solution. Therefore, it can be concluded that no correlation was found between the spectral signature (molecular spectroscopy) of IE-911 and cesium loading.

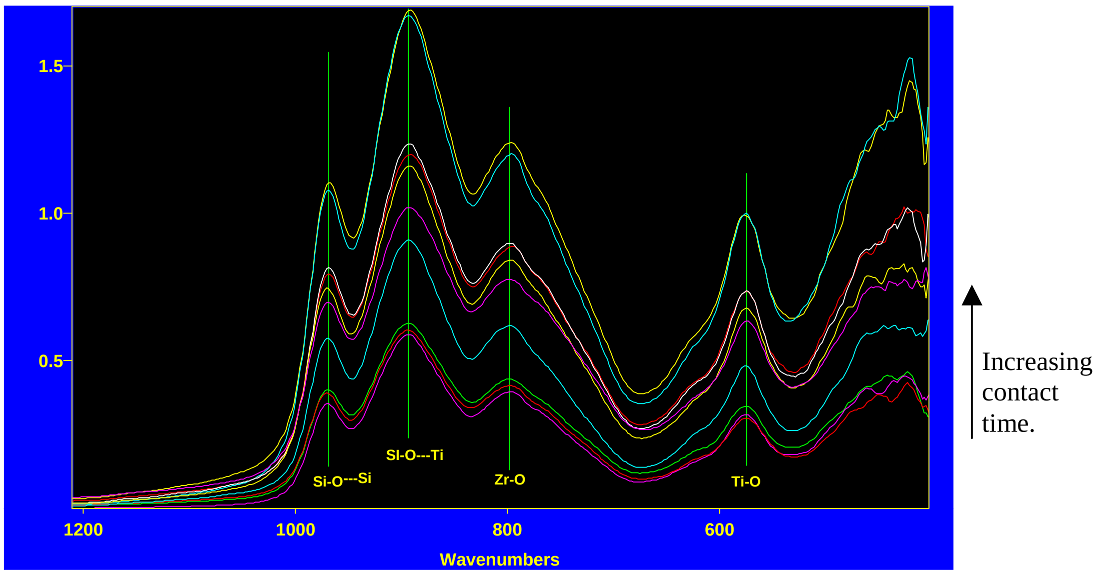

Figure 10. The infrared spectra of IE-911 after contacting "average" salt solution for 10 sec, 20 sec, 40 sec, 2 min., 5 min., 10 min., 20 min., 1 hr., 3 hr., and 5 hr.,

Conclusion

We measured the effective rate of sodium-cesium ion exchange for both IONSIV® IE-910 and IE-911 in "Average" Savannah River Site simulant. The measured values were $2.7 – 5.8 \times 10^{-18}$ m^2/s for IE-910 and $2.5 – 5.8 \times 10^{-11}$ m^2/s for IE-911. The difference in rate of ion exchange between IE-910 and IE-911occurs due to differences in density and particle size, and partial permeation of cesium into IE-911. Analysis of the water content in IE-910 by thermogravimetric techniques yielded an effective cesium diffusivity of 1.75×10^{-19} m^2/s.

Appendix

Cesium Loading Data on IE-910 and IE-911

The amount of cesium loading on IE-910 as a function of time (from Figure 4)

Time (seconds)	$M(t) / M_\infty$
1.44×10^4	.6291
1.44×10^4	.5784
2.88×10^4	.7176
2.88×10^4	.7020
8.64×10^4	.939
8.64×10^4	.9114
1.728×10^5	.957
1.728×10^5	.894
2.592×10^5	1.053
2.592×10^5	.947

The amount of cesium loading on IE-911 as a function of time (from Figure 5)

14400	.6603
14400	.5568
28800	.7358
28800	.704
86400	.9664
86400	.9105
172800	1.05142
172800	.9194
259200	1.1656
259200	.867
604800	.983
604800	1.017

The amount of cesium loading on IE-911 as a function of time (from Figure 8)

Time (seconds)	$M(t) / M_\infty$
1800	0.143109
5400	0.302986
12600	0.444918
19800	0.760553
27000	1

Calculation of the Eigenvalues:

The eigenvalues needed for the diffusion calculation were obtained from the constraint function listed in equation 1. A sample calculation of this function follows.

Ratio of Liquid to solid (LS)	Eigenvalue (q)	tan(eigenvalues)	$3q/(3+LS*q^2)$	difference
0.0845	4.35	2.637598498	2.837603464	-0.2
	4.351	2.645576469	2.837802108	-0.19223
Starting Eigenvalue	4.352	2.653596765	2.838000583	-0.1844
4.35	4.353	2.66165974	2.838198891	-0.17654
	4.354	2.66976575	2.838397031	-0.16863
Increase Eigenvalue by	4.355	2.677915159	2.838595003	-0.16068

0.001	4.356	2.686108332	2.838792808	-0.15268
	4.357	2.694345639	2.838990446	-0.14464
	4.358	2.702627454	2.839187916	-0.13656
	4.359	2.710954156	2.839385218	-0.12843
	4.36	2.719326128	2.839582354	-0.12026
	4.361	2.727743755	2.839779322	-0.11204
	4.362	2.736207431	2.839976123	-0.10377
	4.363	2.74471755	2.840172757	-0.09546
	4.364	2.753274514	2.840369224	-0.08709
	4.365	2.761878727	2.840565524	-0.07869
	4.366	2.770530599	2.840761657	-0.07023
	4.367	2.779230546	2.840957623	-0.06173
	4.368	2.787978985	2.841153423	-0.05317
	4.369	2.796776342	2.841349055	-0.04457
	4.37	2.805623045	2.841544521	-0.03592
	4.371	2.814519528	2.84173982	-0.02722
	4.372	2.823466232	2.841934953	-0.01847
	4.373	2.832463601	2.842129919	-0.00967
	4.374	**2.841512083**	**2.842324719**	**-0.00081**
	4.375	2.850612135	2.842519352	0.008093
	4.376	2.859764217	2.842713819	0.01705
	4.377	2.868968794	2.84290812	0.026061
	4.378	2.878226339	2.843102254	0.035124

The column labeled "eigenvalues" contains a list of "guessed" values. The output from the "Tan (q)" function is subtracted from the output of the function "$3q/(3+SL*q^2)$". The guessed value at which the subtraction is nearly zero and changes sign from "-" to "+", is an eigenvalue. The solid to liquid ratio varied with the type of granular material (i.e. IE-911 versus IE-910). A sample calculation follows.

$\alpha = \frac{V \times M}{v \times m}$; M is the cesium molarity in solution, m is the cesium molarity with respect to CST solid. V is the solution volume while v is the CST volume. After further manipulation the expression for the α can expressed as follows:

$\alpha = \frac{V}{K_d \times \rho \times 0.1g}$; In this expression K_d is the cesium distribution coefficient and ρ stands for the CST density. Typical values for K_d is about 2100 mL/g and ρ is 2.92 g/mL (IE-911).

The total number of eigenvalues chosen for the cesium diffusivity calculation was determined until the value of the eigenvalue was less than 0.001. A sample of the estimate of cesium diffusivity in IE-911 follows.

1	2	3	4	5	6	7
Liquid to Solid Ratio (LS)	Eigen-values (q)	$6SL(SL+1)/(9+9SL+q^2SL^2)$	D x t x $(q/a)^2$	3 x 4	1-Total sum of the entries from column 5	T (=time in seconds)
0.0845	4.37E+00	5.56E-02	1.59E-02	5.47E-02		1.44E+04
	7.52E+00	5.41E-02	4.71E-02	5.16E-02	**6.19E-01**	1.44E+04
	1.06E+01	5.20E-02	9.39E-02	4.74E-02	Compare this	1.44E+04

					number with loading curve	
	1.37E+01	4.95E-02	1.56E-01	4.24E-02		1.44E+04
Guessed diffusivity, “D” (in m^2/s)	1.68E+01	4.67E-02	2.35E-01	3.69E-02		1.44E+04
3.0E-15	1.99E+01	4.37E-02	3.29E-01	3.15E-02		1.44E+04
	2.30E+01	4.06E-02	4.39E-01	2.62E-02		1.44E+04
Radius of IE-911 particle, “a”	2.61E+01	3.76E-02	5.66E-01	2.14E-02		1.44E+04
1.50E-04	2.92E+01	3.47E-02	7.08E-01	1.71E-02		1.44E+04
	3.23E+01	3.20E-02	8.67E-01	1.34E-02		1.44E+04
	3.54E+01	2.94E-02	1.04E+00	1.04E-02		1.44E+04
	3.85E+01	2.70E-02	1.23E+00	7.89E-03		1.44E+04
	4.16E+01	2.49E-02	1.44E+00	5.91E-03		1.44E+04
	4.47E+01	2.29E-02	1.66E+00	4.35E-03		1.44E+04
	4.78E+01	2.11E-02	1.90E+00	3.16E-03		1.44E+04
	5.09E+01	1.95E-02	2.15E+00	2.26E-03		1.44E+04
	5.40E+01	1.80E-02	2.42E+00	1.59E-03		1.44E+04
	5.71E+01	1.67E-02	2.71E+00	1.11E-03		1.44E+04
	6.02E+01	1.54E-02	3.01E+00	7.59E-04		1.44E+04
	6.33E+01	1.43E-02	3.33E+00	5.13E-04		1.44E+04
	6.64E+01	1.33E-02	3.67E+00	3.41E-04		1.44E+04
	6.95E+01	1.24E-02	4.02E+00	2.24E-04		1.44E+04
	7.26E+01	1.16E-02	4.38E+00	1.45E-04		1.44E+04
	7.57E+01	1.09E-02	4.76E+00	9.26E-05		1.44E+04
	7.88E+01	1.02E-02	5.16E+00	5.82E-05		1.44E+04
	8.19E+01	9.54E-03	5.58E+00	3.61E-05		1.44E+04
	8.50E+01	8.97E-03	6.01E+00	2.21E-05		1.44E+04
	8.81E+01	8.44E-03	6.45E+00	1.33E-05		1.44E+04
	9.12E+01	7.96E-03	6.92E+00	7.89E-06		1.44E+04
	9.43E+01	7.51E-03	7.39E+00	4.62E-06		1.44E+04
	9.74E+01	7.10E-03	7.89E+00	2.66E-06		1.44E+04
	1.00E+02	6.72E-03	8.40E+00	1.51E-06		1.44E+04
	1.04E+02	6.37E-03	8.93E+00	8.47E-07		1.44E+04
	1.07E+02	6.04E-03	9.47E+00	4.67E-07		1.44E+04
	1.10E+02	5.74E-03	1.00E+01	2.54E-07		1.44E+04
	1.13E+02	5.46E-03	1.06E+01	1.36E-07		1.44E+04
	1.16E+02	5.20E-03	1.12E+01	7.18E-08		1.44E+04
	1.19E+02	4.95E-03	1.18E+01	3.73E-08		1.44E+04
	1.22E+02	4.73E-03	1.24E+01	1.91E-08		1.44E+04
	1.25E+02	4.51E-03	1.31E+01	9.64E-09		1.44E+04
	1.28E+02	4.31E-03	1.37E+01	4.79E-09		1.44E+04
	1.31E+02	4.13E-03	1.44E+01	2.34E-09		1.44E+04
	1.35E+02	3.95E-03	1.51E+01	1.13E-09		1.44E+04
	1.38E+02	3.79E-03	1.58E+01	5.37E-10		1.44E+04

1.41E+02	3.64E-03	1.65E+01	2.51E-10	1.44E+04
1.44E+02	3.49E-03	1.72E+01	1.16E-10	1.44E+04
1.47E+02	3.35E-03	1.80E+01	5.25E-11	1.44E+04
1.50E+02	3.22E-03	1.87E+01	2.35E-11	1.44E+04
1.53E+02	3.10E-03	1.95E+01	1.03E-11	1.44E+04
1.56E+02	2.99E-03	2.03E+01	4.48E-12	1.44E+04
1.59E+02	2.88E-03	2.11E+01	1.91E-12	1.44E+04
1.62E+02	2.77E-03	2.20E+01	8.03E-13	1.44E+04
1.66E+02	2.68E-03	2.28E+01	3.32E-13	1.44E+04
1.69E+02	2.58E-03	2.37E+01	1.36E-13	1.44E+04
1.72E+02	2.49E-03	2.45E+01	5.44E-14	1.44E+04
1.75E+02	2.41E-03	2.54E+01	2.15E-14	1.44E+04
1.78E+02	2.33E-03	2.64E+01	8.37E-15	1.44E+04
1.81E+02	2.25E-03	2.73E+01	3.21E-15	1.44E+04
1.84E+02	2.18E-03	2.82E+01	1.21E-15	1.44E+04
1.87E+02	2.11E-03	2.92E+01	4.49E-16	1.44E+04
1.90E+02	2.05E-03	3.02E+01	1.64E-16	1.44E+04
1.93E+02	1.98E-03	3.11E+01	5.92E-17	1.44E+04
1.97E+02	1.92E-03	3.21E+01	2.10E-17	1.44E+04
2.00E+02	1.87E-03	3.32E+01	7.33E-18	1.44E+04
2.03E+02	1.81E-03	3.42E+01	2.52E-18	1.44E+04
2.06E+02	1.76E-03	3.53E+01	8.53E-19	1.44E+04
2.09E+02	1.71E-03	3.63E+01	2.84E-19	1.44E+04
2.12E+02	1.66E-03	3.74E+01	9.33E-20	1.44E+04
2.15E+02	1.62E-03	3.85E+01	3.01E-20	1.44E+04
2.18E+02	1.57E-03	3.96E+01	9.58E-21	1.44E+04
2.21E+02	1.53E-03	4.08E+01	3.00E-21	1.44E+04
2.24E+02	1.49E-03	4.19E+01	9.24E-22	1.44E+04
2.28E+02	1.45E-03	4.31E+01	2.80E-22	1.44E+04
2.31E+02	1.41E-03	4.43E+01	8.38E-23	1.44E+04
2.34E+02	1.37E-03	4.55E+01	2.46E-23	1.44E+04
2.37E+02	1.34E-03	4.67E+01	7.13E-24	1.44E+04
2.40E+02	1.31E-03	4.79E+01	2.03E-24	1.44E+04
2.43E+02	1.27E-03	4.92E+01	5.70E-25	1.44E+04
2.46E+02	1.24E-03	5.04E+01	1.57E-25	1.44E+04
2.49E+02	1.21E-03	5.17E+01	4.28E-26	1.44E+04
2.52E+02	1.18E-03	5.30E+01	1.15E-26	1.44E+04
2.55E+02	1.16E-03	5.43E+01	3.02E-27	1.44E+04
2.59E+02	1.13E-03	5.56E+01	7.83E-28	1.44E+04
2.62E+02	1.10E-03	5.70E+01	2.00E-28	1.44E+04
2.65E+02	1.08E-03	5.83E+01	5.03E-29	1.44E+04
2.68E+02	1.05E-03	5.97E+01	1.24E-29	1.44E+04
2.71E+02	1.03E-03	6.11E+01	3.03E-30	1.44E+04

A similar calculation was performed with samples from IE-910 as shown next.

1	2	3	4	5	6	7
Liquid to Solid Ratio (LS)	Eigen-values (q)	6SL(SL+1)/ (9+9SL+q^2SL^2)	D x t x $(q/a)^2$	3 x 4	1-Total sum of the entries from column 5	T (=time in seconds)
0.0845	4.37E+00	5.56E-02	1.54E-02	5.47E-02	**6.14E-01**	1.44E+04
	7.52E+00	5.41E-02	4.56E-02	5.17E-02	Compare this quantity with loading value	1.44E+04
	1.06E+01	5.20E-02	9.10E-02	4.75E-02		1.44E+04
	1.37E+01	4.95E-02	1.52E-01	4.26E-02		1.44E+04
Guessed diffusivity, “D” (m^2/s)	1.68E+01	4.67E-02	2.27E-01	3.72E-02		1.44E+04
1.40E-22	1.99E+01	4.37E-02	3.18E-01	3.18E-02		1.44E+04
	2.30E+01	4.06E-02	4.26E-01	2.66E-02		1.44E+04
IE-910 radius, “a”	2.61E+01	3.76E-02	5.48E-01	2.17E-02		1.44E+04
5.00E-08	2.92E+01	3.47E-02	6.86E-01	1.75E-02		1.44E+04
	3.23E+01	3.20E-02	8.40E-01	1.38E-02		1.44E+04
	3.54E+01	2.94E-02	1.01E+00	1.07E-02		1.44E+04
	3.85E+01	2.70E-02	1.19E+00	8.20E-03		1.44E+04
	4.16E+01	2.49E-02	1.39E+00	6.17E-03		1.44E+04
	4.47E+01	2.29E-02	1.61E+00	4.58E-03		1.44E+04
	4.78E+01	2.11E-02	1.84E+00	3.35E-03		1.44E+04
	5.09E+01	1.95E-02	2.09E+00	2.42E-03		1.44E+04
	5.40E+01	1.80E-02	2.35E+00	1.72E-03		1.44E+04
	5.71E+01	1.67E-02	2.63E+00	1.20E-03		1.44E+04
	6.02E+01	1.54E-02	2.92E+00	8.33E-04		1.44E+04
	6.33E+01	1.43E-02	3.23E+00	5.68E-04		1.44E+04
	6.64E+01	1.33E-02	3.55E+00	3.82E-04		1.44E+04
	6.95E+01	1.24E-02	3.89E+00	2.54E-04		1.44E+04
	7.26E+01	1.16E-02	4.25E+00	1.66E-04		1.44E+04
	7.57E+01	1.09E-02	4.62E+00	1.07E-04		1.44E+04
	7.88E+01	1.02E-02	5.00E+00	6.83E-05		1.44E+04
	8.19E+01	9.54E-03	5.41E+00	4.29E-05		1.44E+04
	8.50E+01	8.97E-03	5.82E+00	2.65E-05		1.44E+04
	8.81E+01	8.44E-03	6.26E+00	1.62E-05		1.44E+04
	9.12E+01	7.96E-03	6.70E+00	9.76E-06		1.44E+04
	9.43E+01	7.51E-03	7.17E+00	5.80E-06		1.44E+04
	9.74E+01	7.10E-03	7.65E+00	3.39E-06		1.44E+04
	1.00E+02	6.72E-03	8.14E+00	1.96E-06		1.44E+04
	1.04E+02	6.37E-03	8.65E+00	1.11E-06		1.44E+04
	1.07E+02	6.04E-03	9.18E+00	6.25E-07		1.44E+04
	1.10E+02	5.74E-03	9.72E+00	3.46E-07		1.44E+04
	1.13E+02	5.46E-03	1.03E+01	1.88E-07		1.44E+04

1.16E+02	5.20E-03	1.08E+01	1.01E-07	1.44E+04
1.19E+02	4.95E-03	1.14E+01	5.36E-08	1.44E+04
1.22E+02	4.73E-03	1.20E+01	2.80E-08	1.44E+04
1.25E+02	4.51E-03	1.27E+01	1.44E-08	1.44E+04
1.28E+02	4.31E-03	1.33E+01	7.30E-09	1.44E+04
1.31E+02	4.13E-03	1.39E+01	3.65E-09	1.44E+04
1.35E+02	3.95E-03	1.46E+01	1.80E-09	1.44E+04
1.38E+02	3.79E-03	1.53E+01	8.72E-10	1.44E+04
1.41E+02	3.64E-03	1.60E+01	4.17E-10	1.44E+04
1.44E+02	3.49E-03	1.67E+01	1.97E-10	1.44E+04
1.47E+02	3.35E-03	1.74E+01	9.13E-11	1.44E+04
1.50E+02	3.22E-03	1.82E+01	4.18E-11	1.44E+04
1.53E+02	3.10E-03	1.89E+01	1.88E-11	1.44E+04
1.56E+02	2.99E-03	1.97E+01	8.36E-12	1.44E+04
1.59E+02	2.88E-03	2.05E+01	3.66E-12	1.44E+04
1.62E+02	2.77E-03	2.13E+01	1.58E-12	1.44E+04
1.66E+02	2.68E-03	2.21E+01	6.71E-13	1.44E+04
1.69E+02	2.58E-03	2.29E+01	2.81E-13	1.44E+04
1.72E+02	2.49E-03	2.38E+01	1.16E-13	1.44E+04
1.75E+02	2.41E-03	2.47E+01	4.70E-14	1.44E+04
1.78E+02	2.33E-03	2.55E+01	1.88E-14	1.44E+04
1.81E+02	2.25E-03	2.64E+01	7.42E-15	1.44E+04
1.84E+02	2.18E-03	2.74E+01	2.88E-15	1.44E+04
1.87E+02	2.11E-03	2.83E+01	1.10E-15	1.44E+04
1.90E+02	2.05E-03	2.92E+01	4.16E-16	1.44E+04
1.93E+02	1.98E-03	3.02E+01	1.54E-16	1.44E+04
1.97E+02	1.92E-03	3.12E+01	5.65E-17	1.44E+04
2.00E+02	1.87E-03	3.22E+01	2.03E-17	1.44E+04
2.03E+02	1.81E-03	3.32E+01	7.22E-18	1.44E+04
2.06E+02	1.76E-03	3.42E+01	2.52E-18	1.44E+04
2.09E+02	1.71E-03	3.52E+01	8.69E-19	1.44E+04
2.12E+02	1.66E-03	3.63E+01	2.95E-19	1.44E+04
2.15E+02	1.62E-03	3.73E+01	9.85E-20	1.44E+04
2.18E+02	1.57E-03	3.84E+01	3.24E-20	1.44E+04
2.21E+02	1.53E-03	3.95E+01	1.05E-20	1.44E+04
2.24E+02	1.49E-03	4.06E+01	3.36E-21	1.44E+04
2.28E+02	1.45E-03	4.18E+01	1.06E-21	1.44E+04
2.31E+02	1.41E-03	4.29E+01	3.27E-22	1.44E+04
2.34E+02	1.37E-03	4.41E+01	9.98E-23	1.44E+04
2.37E+02	1.34E-03	4.52E+01	3.00E-23	1.44E+04
2.40E+02	1.31E-03	4.64E+01	8.88E-24	1.44E+04
2.43E+02	1.27E-03	4.76E+01	2.59E-24	1.44E+04

2.46E+02	1.24E-03	4.89E+01	7.43E-25	1.44E+04
2.49E+02	1.21E-03	5.01E+01	2.10E-25	1.44E+04
2.52E+02	1.18E-03	5.14E+01	5.85E-26	1.44E+04
2.55E+02	1.16E-03	5.26E+01	1.61E-26	1.44E+04
2.59E+02	1.13E-03	5.39E+01	4.34E-27	1.44E+04
2.62E+02	1.10E-03	5.52E+01	1.15E-27	1.44E+04
2.65E+02	1.08E-03	5.65E+01	3.03E-28	1.44E+04
2.68E+02	1.05E-03	5.79E+01	7.81E-29	1.44E+04
2.71E+02	1.03E-03	5.92E+01	1.98E-29	1.44E+04